Matrix Definitions

A *matrix* is a rectangular array of numbers enclosed by a pair of brackets. The brackets may be square [] or rounded ().

For example $\begin{pmatrix} 2 & -1 \\ 1 & 3 \\ 4 & -2 \end{pmatrix}$ and $\begin{bmatrix} 2 & 3 & 0 \\ -1 & 1 & -2 \\ -3 & 0 & 1 \end{bmatrix}$ are matrices.

Dimensions: Rows and Columns

A matrix has a number of *rows* and *columns*.

For example, $\begin{pmatrix} 2 & -1 \\ 1 & 3 \\ 4 & -2 \end{pmatrix}$ has three rows and two columns

and $\begin{bmatrix} 2 & 3 & 0 \\ -1 & 1 & -2 \\ -3 & 0 & 1 \end{bmatrix}$ has three rows and three columns.

The rows×columns defines the *dimensions* of a matrix.

For example $\begin{pmatrix} 2 & -1 \\ 1 & 3 \\ 4 & -2 \end{pmatrix}$ has dimensions 3×2 and can be called a 3×2 matrix.

and $\begin{bmatrix} 2 & 3 & 0 \\ -1 & 1 & -2 \\ -3 & 0 & 1 \end{bmatrix}$ has dimensions 3×3 and can be called a 3×2 matrix.

Vectors

Matrices with just one column (or just one row) are called *vectors*.

For example $\begin{pmatrix} 2 \\ -1 \end{pmatrix}$ and $\begin{bmatrix} 0 & -2 & 3 \end{bmatrix}$ are vectors; $\begin{pmatrix} 2 \\ -1 \end{pmatrix}$ is said to be a *column vector* and $\begin{bmatrix} 0 & -2 & 3 \end{bmatrix}$ is said to be a *row vector*.

Notation

A matrix or vector can be abbreviated by writing them simply as a letter. For a matrix it is conventional to use a capital letter. For a vector it is conventional to use an underlined or bold small letter.

For example we may write $A = \begin{pmatrix} 2 & -1 \\ 1 & 3 \\ 4 & -2 \end{pmatrix}$ or $\underline{x} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$.

For the remainder of this work, we will use rounded brackets and use small letters underlined to denote vectors.

The elements of a vector or matrix can be uniquely addressed using indices.

For example $\underline{x} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$, $\underline{y}$ =$[y_1 \quad y_2 \quad y_3]$ and $A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{pmatrix}$.

Square and diagonal matrices

A matrix with the same number of rows as columns is said to be a *square* matrix.

For example $\begin{pmatrix} 2 & 3 & 0 \\ -1 & 1 & -2 \\ -3 & 0 & 1 \end{pmatrix}$ is a square matrix.

A square matrix that is zero everywhere apart from on the diagonal is said to be a *diagonal matrix*.

For example $\begin{pmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$is a diagonal matrix.

The transpose of a matrix

By reflecting a matrix across the diagonal we obtain its *transpose* and is denoted by a T superscript.

For example the transpose of $B = \begin{pmatrix} 2 & 3 & 0 \\ -1 & 1 & -2 \\ -3 & 0 & 1 \end{pmatrix}$ is $B^T = \begin{pmatrix} 2 & -1 & -3 \\ 3 & 1 & 0 \\ 0 & -2 & 1 \end{pmatrix}$

and the transpose of $A = \begin{pmatrix} 2 & -1 \\ 1 & 3 \\ 4 & -2 \end{pmatrix}$ is $A^T = \begin{pmatrix} 2 & 1 & 4 \\ -1 & 3 & 2 \end{pmatrix}$.

Further Work and Exercises

The accompanying worksheet[1] has exercises on matrix definitions. The worksheet by PPlato[2] provides further information and exercises on matrix definitions. Mathcentre includes an introduction the matrices[3] and further explanation and exercises[4].

---

[1] Matrix Definitions Exercises
[2] Matrices (Pplato)
[3] Matrices - what is a matrix? (Mathcentre)
[4] Symmetric matrices and the transpose of a matrix